Halos and undecidability of tensor-stable positive maps

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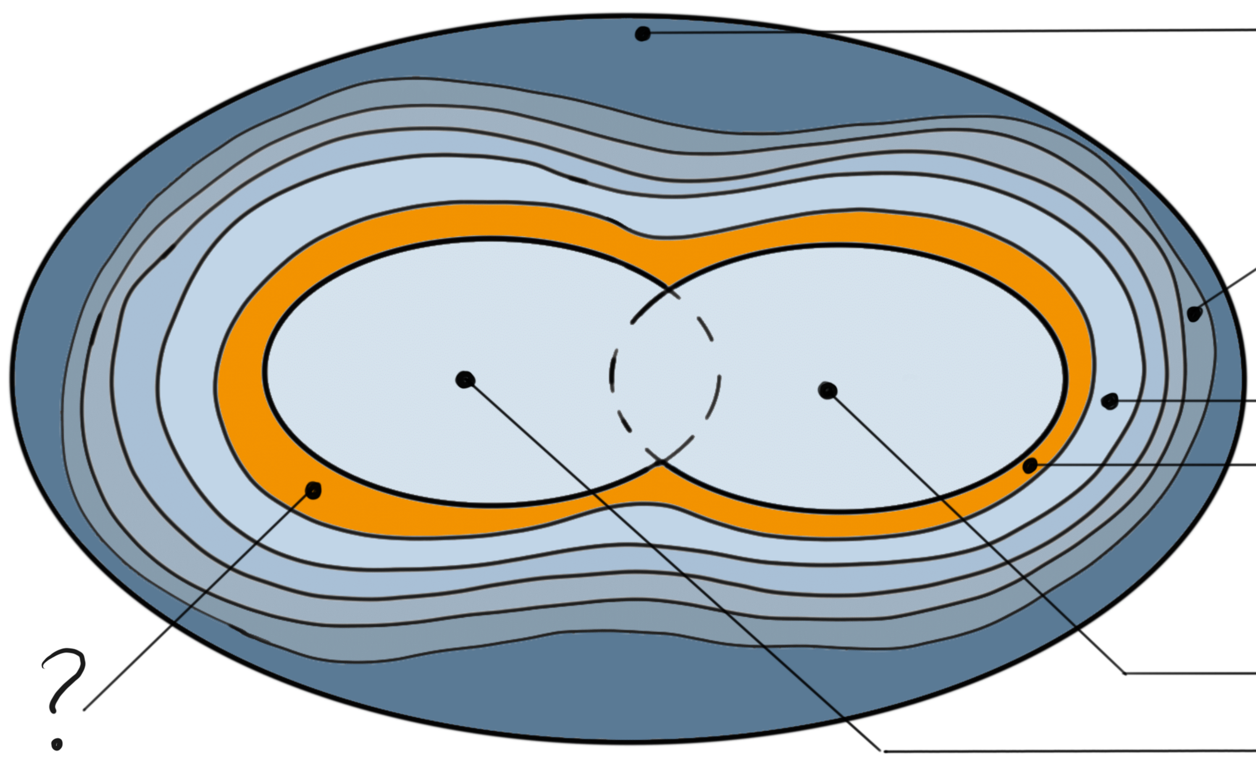
Short teaser video:

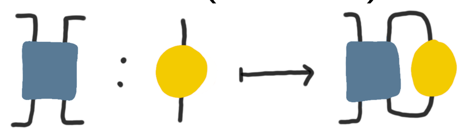
Paper: arXiv:2110.02113

Do there exist non-trivial tensor-stable positive maps?

Positive linear maps $\mathcal{P} : \mathcal{M}_{d_1} \to \mathcal{M}_{d_2}$



Positive ($\mathcal{P} \succcurlyeq 0$) : if $X \geqslant 0$ then $\mathcal{P}(X) \geqslant 0$



2-tensor-stable positive: $\mathcal{P} \otimes \mathcal{P} \succcurlyeq 0$

n-tensor-stable positive : $\mathcal{P}^{\otimes n} \succcurlyeq 0$

Tensor-stable positive (tsp) : $\mathcal{P}^{\otimes n} \succcurlyeq 0$ for all $n \in \mathbb{N}$

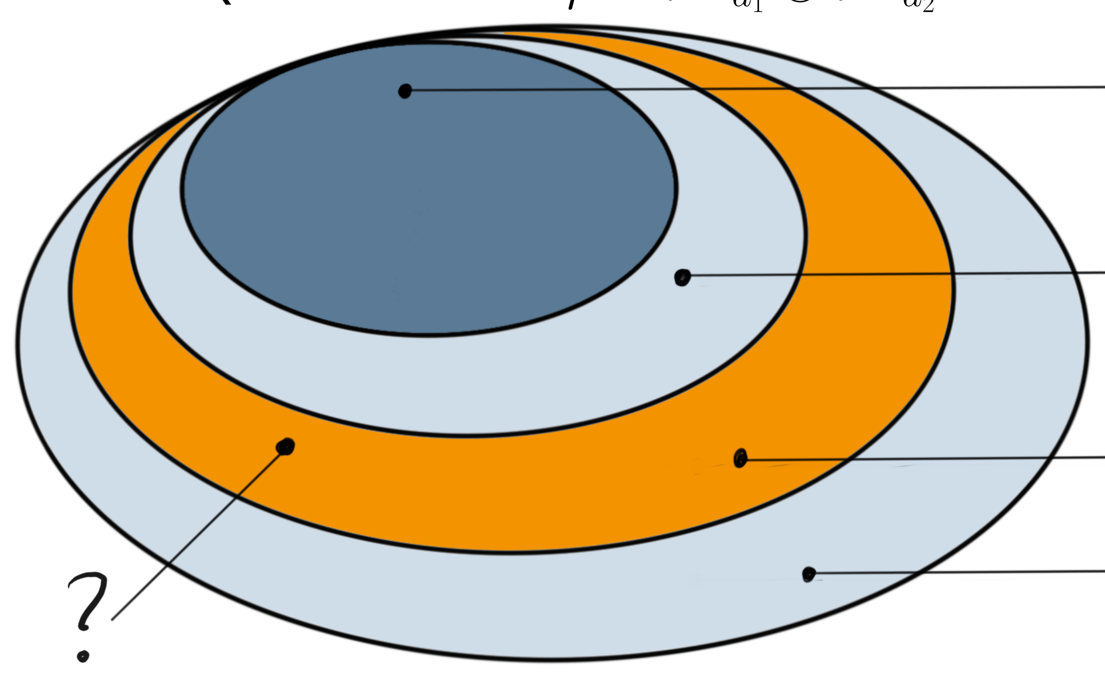
Trivial tsp maps:

Completely positive (cp): $\mathrm{id}_d \otimes \mathcal{P} \succcurlyeq 0$ for all d

Completely co-positive: $\mathcal{P} = T \circ \mathcal{S}$, with T transposition and $\mathcal{S}$ cp

Do all undistillable quantum states have a positive partial transpose?

Quantum states $\rho \in \mathcal{M}_{d_1} \otimes \mathcal{M}_{d_2}$



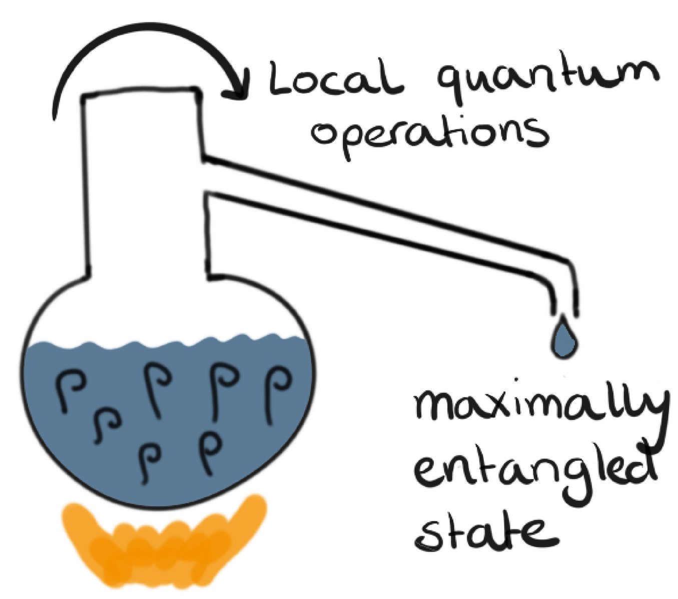
Separable:
$\rho = \Sigma_i \ \alpha_i \otimes \beta_i$ with $\alpha_i, \beta_i \geqslant 0$

Positive Partial Transpose (PPT):
$\rho^{T_2} = (\mathrm{id}_{d_1} \otimes T_{d_2})\rho \geqslant 0$

Undistillable

NPT

Distillation:

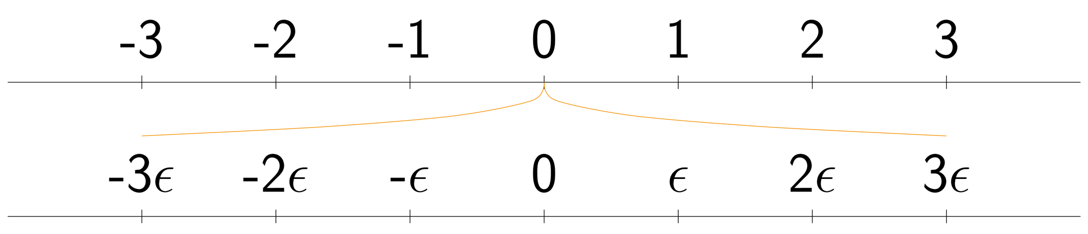


Connection

Existence of non-trivial tensor-stable positive maps $\mathcal{P} : \mathcal{M}_{d_1} \to \mathcal{M}_{d_2}$, implies NPT undistillable states in $\mathcal{M}_{d_1} \otimes \mathcal{M}_{d_1}$ and $\mathcal{M}_{d_2} \otimes \mathcal{M}_{d_2}$ (1).

Halos

We consider the problem on the hyperreals. The hyperreals ${}^*\mathbb{R}$ have additional infinitesimal elements ϵ.



Definition hypercomplex: ${}^*\mathbb{C} = {}^*\mathbb{R} + i{}^*\mathbb{R}$.

Theorem 1: Non-trivial tsp on ${}^*\mathbb{C}$

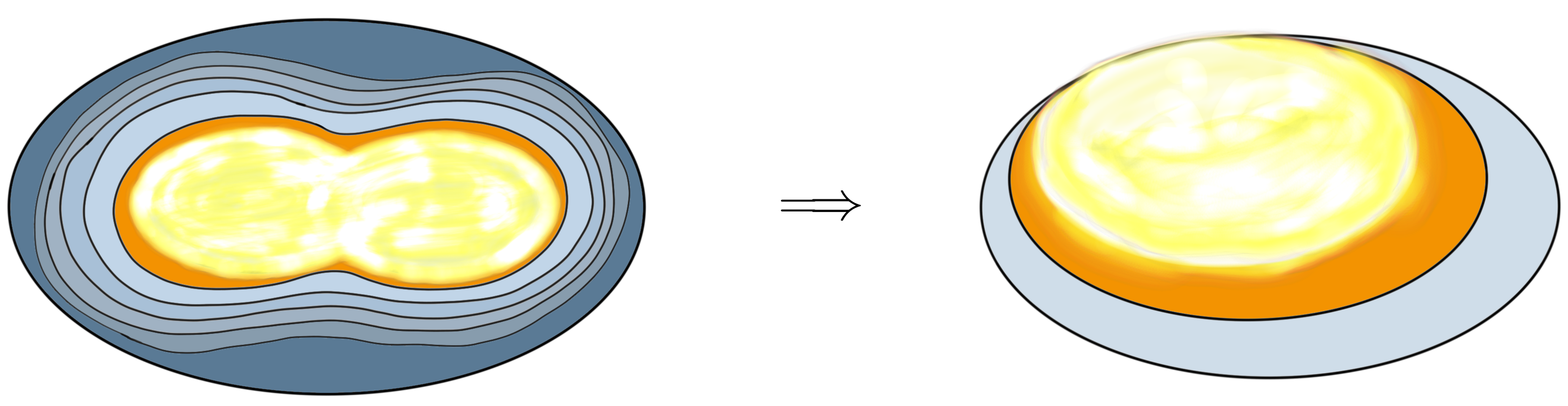
There exist non-trivial tensor-stable positive maps $\mathcal{P} : \mathcal{M}_{d_1}({}^*\mathbb{C}) \to \mathcal{M}_{d_2}({}^*\mathbb{C})$ on the hypercomplex.

Idea of proof: consider $\text{trivial map} - \epsilon \cdot \text{identity map}$.

When transferred back to $\mathbb{C}$, $\epsilon = 0$ so the maps are trivial again.
The connection to quantum still holds on ${}^*\mathbb{C}$. Therefore:

Theorem 2: Hyperquantum states

There exist NPT undistillable hyperquantum states on ${}^*\mathbb{C}$.



Theorem 3: Non-trivial tsp on ℓ^2

There exist non-trivial tsp maps $\mathcal{P} : \mathcal{M}_{d_1}(\ell^2(\mathbb{C})) \to \mathcal{M}_{d_2}(\ell^2(\mathbb{C}))$.

Where positivity is defined as: $(x_n) \geq 0 \in \ell^2$ if $\{n : x_n \geq 0\} \in \text{ultrafilter}$.

Undecidability

Definition matrix-multiplication tensor (mamu):

$$|\chi_n\rangle = \sum_{i_1,\ldots,i_n=1}^{d} |i_1, i_2\rangle \otimes |i_2, i_3\rangle \otimes \cdots \otimes |i_n, i_1\rangle$$

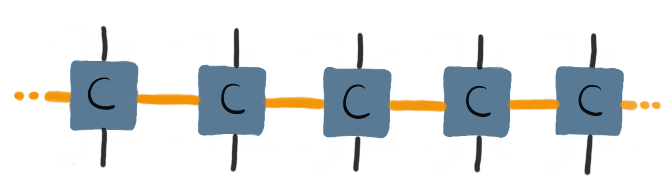
Problem mamu: Given d and a linear map $\mathcal{P} : \mathcal{M}_d \to \mathcal{M}_d$, is $\mathcal{P}^{\otimes n}(|\chi_n\rangle\langle\chi_n|) \geqslant 0$ for all n?

Theorem 4: Undecidability

Problem mamu is undecidable.

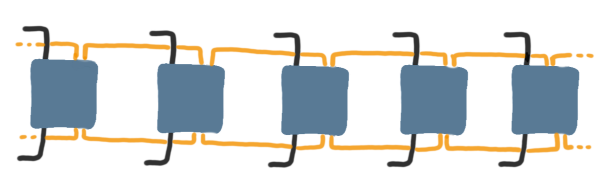
Idea of proof:

Undecidable problem MPO (2): Given a Matrix Product Operator defined by C_i, is $\tau_n(C) \geqslant 0$ for all n?



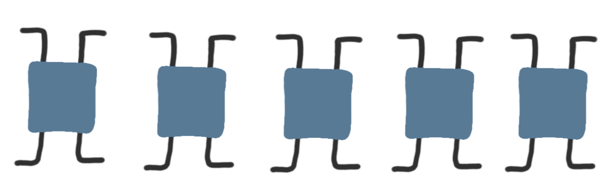
Reduction ✔

Problem mamu



Reduction ✗

Problem TSP: Given d and a linear map $\mathcal{P} : \mathcal{M}_d \to \mathcal{M}_d$, is $\mathcal{P}^{\otimes n} \succcurlyeq 0$ for all n?



Proving undecidability of Problem TSP would imply existence of NPT undistillable states.

(1) A. Müller-Hermes, D.Reeb, and M. M. Wolf, Positivity of linear maps under tensor powers. J. Math. Phys. **57**, 015202 (2016).
(2) G. De las Cuevas, T. S. Cubitt, J. I. Cirac, M. M. Wolf, and D. Pérez-García, Fundamental limitations in the purifications of tensor networks. J. Math. Phys. **57**, 071902 (2016).

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