

7Whi gVXi

- 3-YHĤ, Nasymmetric %syLH "L-%o%e sp Ž >Y%6~ ~, %lo
- 5pL "L-%o" , HL %o%oBs>"LH", >^>-H, } } >^>B ~%oYB'LHN, } contractions, N"pL "L-%o" E sp s%6nY > ĉLB', %o
- flæ>B' Bp>>B'L's > , ~, N"pL > } , %o%L s s%oN"pL "L-%o" %oNĤ > ĉ>Yl >-H>sn~} L~"%o" HL^sLHs "pL high-dimensional regime

7hnb b ZigX gVc` #1 he ^ ZY gVcYdb i Zchdgh

ĉ, ~%HL "pL non-symmetric %syLH "L-%o" , HL

$$T = \frac{1}{n} \sum_{i=1}^n \frac{x_i x_i^T}{\|x_i\|^2} + \frac{1}{n} \sum_{i=1}^n \frac{x_i x_i^T}{\|x_i\|^2} + \text{Noise}$$

E pL^L (x_1, ..., x_d) S^{n-1} x_1, ..., x_d N(0, 1) ssH >-H n = P_{i=1}^d n_i
 5pL ...>} L^L > 0 B, ~^, %pL %a~> , ~, %lo > , 3* 2

(A1) Growth rate assumptions: %o_i P_{j=1}^d n_j c_i (0, 1) >-H d = O(1)

I Zchdghĉ\j aVgkVj Zh VcY kZXidgh

) >æš Y} syL q, , HL%o> , ^) &fl , NAL%o>-y 1 >...ĉ, æš > , ~, NT

$$\arg \min_{u_1, \dots, u_d} T(u_1, \dots, u_d)$$

flĤYs>L~" , &š

$$\max_{u_i=1} |T(u_1, \dots, u_d)|$$

5pL %%5 B, -Hs , ~%L^sLHAY "pL %o , ~>Y...s"% (u_1, ..., u_d) >L i [d]

$$T(u_1, \dots, u_{i-1}, u_{i+1}, \dots, u_d) = u_i >-H u_i = 1$$

Goal: 7-HL^ (A1) p, E , Læ...L%o%pL >%æ ... , B%oN >-H x_i, u_i , N"pL) &fl

7hhdX^viZY gVcYdb b Vigmb dYZaĉd = 3Ž

Stein's Lemma: &L" X N(0, 1) E[Xf(X)] = E[f'(X)]

7-HL^ (A1) concentrates >, Y-Hs%bæ.LB^> , ~>-HE sp 3^Ls-%&L } >

$$E = \frac{1}{n} \sum_{i,j,k} u_{ij} u_{jk} \frac{u_{ij}}{X_{ijk}} + \frac{1}{n} \sum_{i,j,k} u_{ij} u_{jk} \frac{u_{jk}}{X_{ijk}} + \frac{1}{n} \sum_{i,j,k} u_{ij} u_{jk} \frac{u_{ij}}{X_{ijk}}$$

Resolvent matrix: R(z) = (3(T, u_1, u_2, u_3) - zI_n)^{-1}

%o "pL ~, ~ ĉ>-sps-n "L" %oĉ, ĉL traces , NR(z)

$$\frac{u_{ij}}{X_{ijk}} - \frac{1}{n} u_{ij} u_{jk} R_{jj}^2(z), \frac{u_{ij}}{X_{ijk}} - \frac{1}{n} u_{ij} u_{jk} R_{jj}^2(z), \frac{u_{ij}}{X_{ijk}} - \frac{1}{n} u_{ij} u_{jk} R_{kk}^3(z)$$

i i Ža ZhigVch[dgh

Definition Ž sL~>... A>AssY} L>%L "pL 3 L "W%o>-%Ĭ" , N %HL-LH>%o

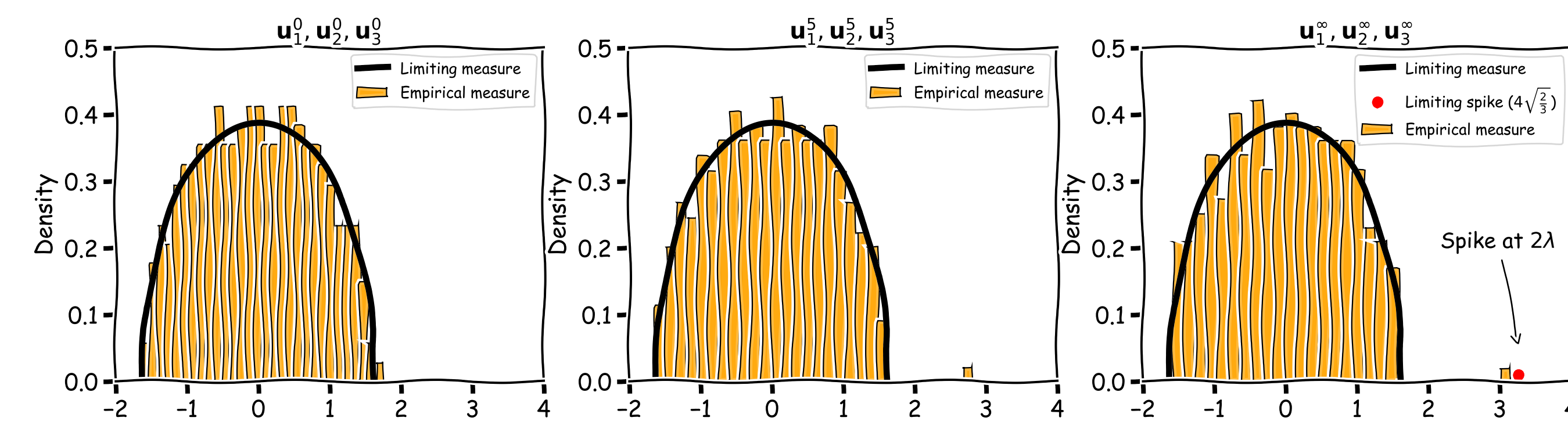
$$g(z) = \frac{z d(z)}{-z}$$

Ž sL~> n x n %o } L^B } >^>S >-H HL~ , "L i s%L sL~>Yl%o" pL empirical spectral measure fl3) , NS >-H "pL B, ^L%o -Hs-n 3 L "W%o>-%Ĭ" } Læ...L%o%o

$$s = \frac{1}{n} \sum_{i=1}^n \frac{1}{1-z} = \frac{1}{n} \text{tr} R_S(z), z \in \mathbb{C} \setminus S(s)$$

E pL^L R_S(z) = (S - zI_n)^{-1} %o-H%oĬ "pL ^L%oĉL~" , NS

Bĉ ĉ\ heZXigVab ZVhj gĉ d[d(T, u_1, ..., u_d)



Ĥ snY^L 3.LB^Y} , N 3(T, u_1, u_2, u_3) >^>S L^> , ~%o, 5, , N... E L^sL^> , ~ >...sLH, ~ T n_1 = n_2 = n_3 = 100 >-H = 0

(A2) (u_1, ..., u_d) %Bp "p>" %o, " >-LsL~>Yl , N d(T, u_1, ..., u_d)

Theorem 1 7-HL^ (A1) >-H (A2) "pL fl3) , N d(T, u_1, ..., u_d) B~ĉL^nl%o, > deterministic measure E p, %o 3 L "W%o>-%Ĭ" } %oĉsL~>%g(z) = \sum_{i=1}^d g_i(z) %Bp "p>" [g(z)] > 0 N^ [z] > 0 E sp

$$\frac{1}{n} \text{tr} R^{ii}(z) \geq g_i(z) = \frac{g(z) + z}{2} - \frac{4c_i + (g(z) + z)^2}{2}, z \in \mathbb{C} \setminus S(z)$$

Corollary 1 "Nc_i = \frac{1}{d} N^ > i [d] "pL~"pL fl3) , N d(T, u_1, ..., u_d) B~ĉL^nl%o, > semi-circle Hs%oAY , ~%o...~"LH, ~ S(z) = -2 \frac{d-1}{d}, 2 \frac{d-1}{d} E sp

$$(dx) = \frac{d}{2(d-1)} \frac{4(d-1)}{d} \frac{1}{-x^2}, g(z) = \frac{-zd + d}{2(d-1)} \frac{z^2 - \frac{4(d-1)}{d}}{2(d-1)}, z \in \mathbb{C} \setminus S(z)$$

7hnb eidi X hĉ\j aVgkVj Z VcY Va^cb Zcih

Theorem 2 7-HL^ (A1) >-H (A2) N^ d 3 "pL^L Læ...s%o s > 0 %Bp "p>" N^ > s

$$\frac{1}{n} \sum_{i=1}^d \frac{1}{1-z} \geq \frac{1}{n} \sum_{i=1}^d \frac{1}{1-z} \geq \frac{1}{n} \sum_{i=1}^d \frac{1}{1-z}$$

E pL^L %o %d%ĉf() = 0 E sp f(z) = z + g(z) - \sum_{i=1}^d g_i(z) >-H

$$g_i(z) = \frac{1}{j=i} \frac{1}{j(z)}, i(z) = \frac{1}{z + g(z) - g_i(z)}$$

E pL N^ [0, s] %A, Y-HLHAY > B, ~%o~" >-H / x_i, u_i | \geq %o 0

Corollary 2 ^YAB "L-%o%o" Nd = 3 E sp c_i = \frac{1}{3} N^ > \frac{2}{3}

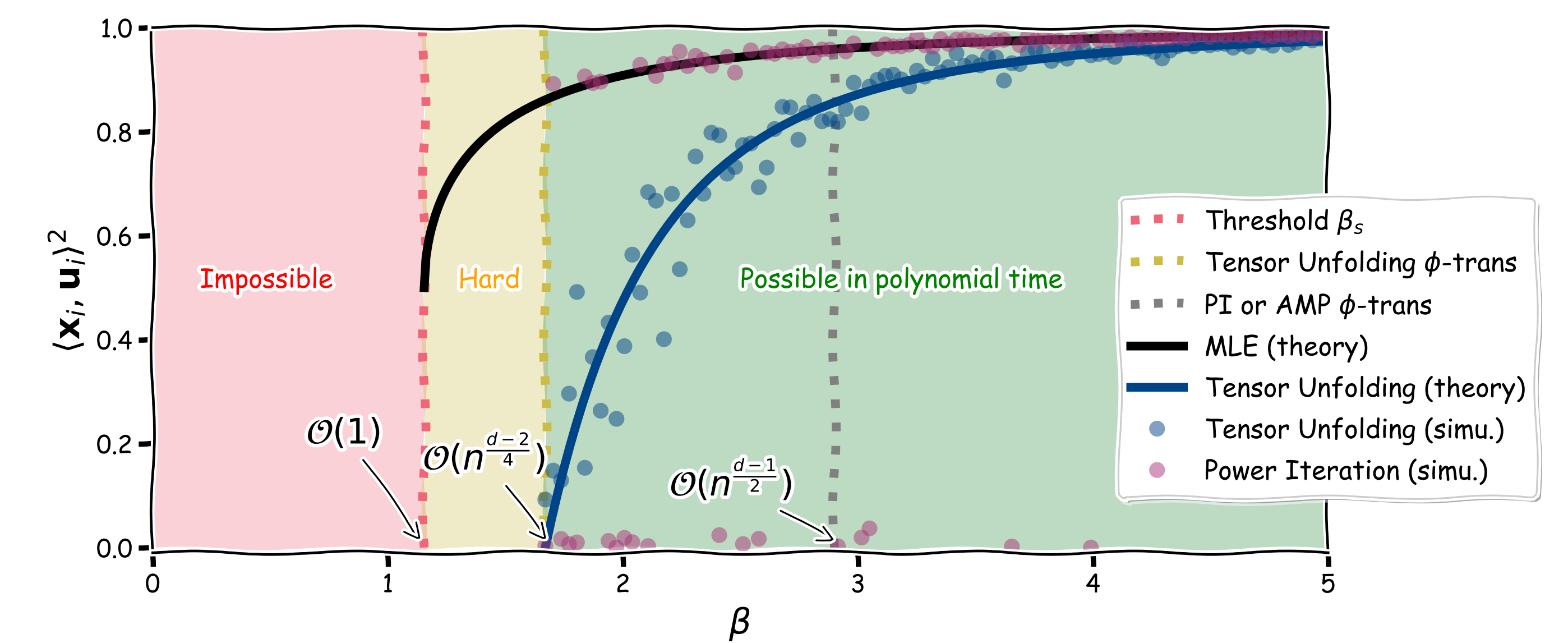
$$\frac{1}{n} \sum_{i=1}^d \frac{1}{1-z} = \frac{1}{n} \sum_{i=1}^d \frac{1}{1-z} = \frac{1}{n} \sum_{i=1}^d \frac{1}{1-z}$$

Corollary 3 3.syLH^>-H, } } >^>B L%o "Nd = 3 E sp c_1 = c >-H c_2 = 1 - c N^ %o} L c (0, 1) "pL %syLH "L-%o" , HL ALB } L%o spiked matrix model sL c_3 = 0

$$\frac{1}{n} \sum_{i=1}^d \frac{1}{1-z} = \frac{1}{n} \sum_{i=1}^d \frac{1}{1-z} = \frac{1}{n} \sum_{i=1}^d \frac{1}{1-z}$$

E pL N^ [0, s] \geq %o \frac{1}{1+2} \frac{1}{c(1-c)} >-H / x_i, u_i | \geq %o 0

I] ZdgnkZghj hĉb j aVi ^dch



Ĥ snY^L 5p^L%o, H%o-H>%æ ... , B>sn~} L~%Ĭ^ >BYAB %syLH^>-H, } "L-%o") &fl in black >-H "L-%o" Y-N Hs-n "L~" , Y%o s-A YL 3š Y > , ~%E sp ... E L^sL^> , ~ >-H "L-%o" Y-N Hs-n >...sLH, ~ >BYAB "L-%o" E sp n_i = 70

9dcXj hĉc

- 5pL HL^sLH^L%o" seems , HL%oAL "pL Alp>ĉs^ , N"pL) &fl
- 3 Y-BL>^p, E , Bp>>B'L'sL "pL phase transition N^ "pL) &fl E sp , Y^ >...ĉ, >Bp >%o%o } L~ , ~LHs- Ž , Y>^" N^ %o } L^B "L-%o%o
- Universality >-H nL~L^>š > , ~ , higher-ranks ^L } >s " , s-ĉL%oĉL

HZ[ZgĉcXZh

&š &ly Ž L-n 3s-nY > ĉ>Yl%o-HLsL~>Yl%oN "L-%o%o> ĉ>^s , ~ >...ĉ, >Bp %o"ffifi "L~> , ~ > , y%æ ... , ~ } .Y"> , ~ > Hĉ>-BL%o-) Y 3L~%o" H>...ĉL / , BL%oĉn "ffifi

HL) , ^s%ž , Y>^" \$ %oŽ L~sYl 2, } >s^ , YsL^ >-H / s^L^ , } , ~ ^ 2>-H, }) >^> /L^%dLB ĉL , ~ 2>-H, } 5L~%o%o%o" L~" , Y%ož ^>H / >-s = pL-nYĤ Ž Y>-n >-Hs, Y>-n Ž Y>-n & -n 2>-H, }) >^>BL%o >-HBL~%o 7-N Hs-n > ; sL...s"%o > ; s

